Nonparametric Sensitivity Analysis for Randomized Experiments with Missing Outcomes*

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Abstract

In randomized experiments, the outcome can be missing due to non-response or attrition. Although the conditional independence assumption yields point identification of causal effects, the assumption can be violated if people who respond and who do not are not comparable due to the presence of unmeasured confounders. This paper proposes a method to construct a confidence interval for treatment effects that accounts for the possible violation of the conditional independence assumption. The proposed confidence interval is derived based on a nonparametric inequality-based sensitivity analysis that bound the ratio of quantiles of the potential outcomes. The proposed method improves upon the existing methods in that it is applicable beyond the average treatment effect such as the quantile treatment effect, and do not impose restrictions on the type of the outcome. I apply the proposed method to two randomized experiments where the outcome is missing either due to panel attrition or due to the presence of the "don't know" option.

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1 Introduction

In randomized experiments, practitioners often encounter data where the outcome is missing for a subset of experimental subjects. Missingness could occur for a variety of reasons such as the attrition from the follow-up survey, or non-response to a particular question designed to measure the outcome in the post-treatment survey. It is well understood that unless the missingness is completely independent of the potential outcome, the analysis based on the complete observations will produce biased estimates for causal effects.

The standard practice to address this problem relies on an assumption that the missingness is independent of the potential outcome given the baseline covariates within the treatment condition (e.g., Robins, Rotnitzky and Zhao, 1995; Rubin, 1976). Under the conditional independence assumption, the causal effects such as the average treatment effect are nonparametrically identified, and the consistent estimators are available via the weighting or the regression method (e.g., Tsiatis, 2007). However, the conditional independence assumption can be a strong assumption in practice. For example, collecting many baseline covariates is difficult in some experiments because of the cost or ethical issues. In such a case it is possible that unmeasured characteristics are associated both with the outcome and the missingness, which results in a biased estimate and inference.

This paper proposes a sensitivity analysis and sensitivity-aware confidence interval that does not impose the conditional independence assumption. In the proposed approach, scholars instead assume that the ratio of quantiles between the observed and missing distributions are bounded by a pre-specified constant, which serves as the sensitivity parameter. Since the conditional independence assumption essentially imposes that two distributions are identical given covariates, the proposed framework incorporates the standard assumption as a special case. I show that the average treatment effect and quantile treatment effects can be bounded as a function of the sensitivity parameter and the marginal distribution of the observed outcome.

This paper makes two methodological contributions. First, the proposed method is applicable to estimands beyond the average treatment effects (ATE). Contrary to the existing methods that are specific to ATE (e.g., Blackwell, 2014; Coppock et al., 2017; Egami, 2020; Horowitz and Manski, 2000, among others), the proposed method is applicable to any estimand that is a function of marginal distributions of potential outcome. In the paper, I derive a bound on the quantile treatment effect (QTE) as an application of the proposed method. Second, the proposed method does not require a specific type of outcome. For example, Imai (2009) proposes an identification strategy for ATE in the presence of non-ignorable missing outcome when the outcome is binary. While such methods are useful when data meets specific requirements, scholars can apply the proposed method whether the outcome is either binary, ordinal, or continuous.

2 The Proposed Methodology

In this section, I describe the proposed methodology. Section 2.1 introduces the notation and presents the identification formula under the conditional independence assumption, which will be used as the basis to derive the proposed sensitivity analysis. In Section 2.2, I introduce the inequality-based approach to bound the treatment effects. Specifically, I derive bounds on the average treatment effect and the quantile treatment effect under the assumption that the ratio of two distribution functions are bounded. Based on these identification bounds, Section 2.3 describes how to construct a confidence intervals for the causal effects.

2.1 The Setup

Consider a randomized experiment with binary treatment $D_i \in \{0, 1\}$ for $i \in \{1, \ldots, n\}$. We observe the outcome $Y_i \in \mathcal{Y} \subseteq \mathbb{R}$ as well as covariates $\mathbf{X}_i \in \mathcal{X} \subseteq \mathbb{R}^p$. I consider a scenario that the outcome is missing for some of the observations. Let $R_i = \mathbf{1}\{Y_i \neq \text{NA}\}$ denote a response indicator that takes 1 if the outcome is observed for unit i, and takes 0 if the outcome is missing.

Let $Y_i(d)$ denote the potential outcome for condition $d \in \{0, 1\}$. In this paper, I focus on two popular causal esitmands: the average treatment effect (ATE), which is defined as

$$\tau = \mathbb{E}[Y_i(1) - Y_i(0)] \tag{1}$$

and the quantile treatment effect (QTE), which is defined as

$$\psi(\alpha) = Q_{Y(1)}(\alpha) - Q_{Y(0)}(\alpha). \tag{2}$$

where $Q_{Y(d)}(\alpha)$ is the quantile function of $Y(d)$ evaluated at $100 \times \alpha$-th percentile.

Throughout the paper, I assume that the treatment is randomized. The extension to the observation studies under the selection-on-observable is presented in Appendix B.

Assumption 1 (Randomized Experiment)**.** $Y_i(d) \perp\!\!\!\perp D_i$ *for* $d \in \{0, 1\}$.

In general, the complete case analysis that compares observed responses does not yield the identification without a strong assumption of missing completely at random (i.e., the missing indicator is independent of the potential outcome). Instead, the standard approach is to posit the conditional independence. Let $R_i(d)$ denote the potential response indicator for $d \in \{0, 1\}$,

Assumption 2 (Sequential Attrition Ignorability)**.** $Y_i(d) \perp\!\!\!\perp R_i(d) \mid \mathbf{X}_i, D_i = d$ *for* $d \in \{0, 1\}$.

Assumption 2 requires that within the treatment strata, the potential outcome is independent of the response indicator. Note that Assumption 2 allows for the differential attrition, or more generally that the treatment can affect the response indicator.

Under Assumption 1 and 2, the marginal distribution of the potential outcome is identified (Result 1).

Result 1 (Identification)**.** *Suppose that Assumption 1 and 2 hold. Then, the distribution of the potential outcome is identified as*

$$\mathbb{P}(Y(d) \leq y) = \int_{\mathcal{X}} \mathbb{P}(Y \leq y \mid D = d, R = 1, \mathbf{X} = \boldsymbol{x}) dF_{\mathbf{X}}(\boldsymbol{x}).$$

The result follows by the law of iterated expectation, and Assumption 1 and 2. Though the result is standard, the proof is given in Appendix A.2 for completeness.

With the above result, we can estimate the causal effect by adjusting for observed covariates. Let $e_d(\mathbf{X}_i) = \Pr(R_i = 1 \mid \mathbf{X}_i, D_i = d)$ denote the attrition score. Then, the estimator for ATE is given in the IPW form as

$$\widehat{\tau}_{\texttt{IPW}} = \frac{1}{n_1} \sum_{i=1}^{n} \frac{D_i R_i Y_i}{\widehat{e}_1(\mathbf{X}_i)} - \frac{1}{n_0} \sum_{i=1}^{n} \frac{(1 - D_i) R_i Y_i}{\widehat{e}_0(\mathbf{X}_i)} \tag{3}$$

where $n_d = \sum_{i=1}^{n} \mathbf{1}\{D_i = d\}$. Other forms of estimators such as the outcome regression or the augmented IPW are also available in the literature. Similarly, QTE can be estimated via the weighted quantile regressions where the weights are the inverse of the attrition score (e.g., Firpo (2007); Powell (2020); Zhang and Zheng (2020)).

2.2 Bounds on Treatment Effects

The ignorability assumption (Assumption 2) provides an identification of the treatment effects. However, the assumption can be quite strong in practice: there are several scenarios under which the assumption can be violated. For example, it is possible that scholars cannot observe a rich set of baseline covariates. In a field experiment, for example, it is costly and sometimes difficult to obtain beyond the baseline demographic variables about experimental subjects. In such case, a researcher might worry about the presence of unobserved confounders that affect both the missingness and the outcome itself. In another scenario, it is possible that non-response is due to the outcome variable itself. This is a typical example of missing not at random, and can happen when the value of the outcome is sensitive.

Instead, I propose a method that enables scholars to draw inference on treatment effects without imposing the ignorability assumption. The first step toward the goal is to obtain bounds on the treatment effects. Following the tradition of imposing restrictions on the selection bias, I propose bounding the ratio of two distributions of the potential outcomes. Specifically, I assume that the distribution of the potential outcome for the observed $R_i = 1$ and the distribution for the missing $R_i = 0$ are bounded above by ζ and below by $1/\zeta$:

$$\frac{1}{\zeta} \leq \frac{\mathbb{P}(Y \leq y \mid R = 1, \mathbf{X} = \boldsymbol{x}, D = d)}{\mathbb{P}(Y \leq y \mid R = 0, \mathbf{X} = \boldsymbol{x}, D = d)} \leq \zeta, \quad \forall (d, \boldsymbol{x}) \in \{0, 1\} \times \mathcal{X} \tag{4}$$

where $\zeta \geq 1$. Note that the denominator of the expression is the quantity that we do *not* observe (i.e., the distribution of the outcome for the non-response observations). When $\zeta = 1$, we recover the ignorability

assumption (Assumption 2), because it implies $\mathbb{P}(Y \leq y \mid R = 1, \mathbf{X} = \boldsymbol{x}, D = d) = \mathbb{P}(Y \leq y \mid R = 0, \mathbf{X} = \boldsymbol{x}, D = d)$. To see what the above inequality implies when $\zeta \neq 1$, consider $\zeta = 1.2$. This value of ζ implies that for a value of y that is the median under the observed distribution, the same value of y should be between $50/1.2 \approx 42$nd and $50 \times 1.2 = 60$th percentile under the missing distribution.

Given the knowledge of ζ, the above inequality suggests the following bounds on the distribution function for the missing observations,

$$\mathbb{P}(Y \leq y \mid R = 0, \mathbf{X} = \boldsymbol{x}, D = d) \leq \zeta\, \mathbb{P}(Y \leq y \mid R = 1, \mathbf{X} = \boldsymbol{x}, D = d)$$
$$\mathbb{P}(Y \leq y \mid R = 0, \mathbf{X} = \boldsymbol{x}, D = d) \geq \frac{1}{\zeta}\, \mathbb{P}(Y \leq y \mid R = 1, \mathbf{X} = \boldsymbol{x}, D = d)$$

for $y \in \mathcal{Y}$.[1]

Therefore, the proposed inequality-based approach enables researchers to bound the distribution of the unobserved outcome (i.e., the outcome among missing observations) using the information about the distribution of the observed outcome. Since causal quantities such as the average treatment effect or the quantile treatment effect are a function of the observed as well as unobserved outcome, having partial information about the unobserved outcome allows us to partially identify causal effects.

I first consider a derivation of the identification bound for ATE. The key idea for bounding ATE is that the mean of a random variable can be written as a integral of the cumulative distribution function. Specifically, I use the identify in Lemma 1 to bound ATE.

Lemma 1 (Expectation of X)**.** *Consider a random variable $X \in \mathcal{X} \subseteq \mathbb{R}$ with the cumulative distribution function $\mathbb{P}(X \leq x)$. Then, the expectation of X can be written as*

$$\mathbb{E}[X] = \int_{\{x \geq 0\} \cap \mathcal{X}} \big\{1 - \mathbb{P}(X \leq x)\big\} dx - \int_{\{x < 0\} \cap \mathcal{X}} \mathbb{P}(X \leq x) dx.$$

With Lemma 1, we can bound the mean of the outcome among the missing observations. For example, the upper bound of the outcome among the missing is given by

$$\begin{aligned}
&\mathbb{E}[Y \mid R = 0, \mathbf{X} = \boldsymbol{x}, D = d] \\
&\quad = \int_0^\infty \big[1 - \mathbb{P}(Y \leq y \mid R = 0, \mathbf{X} = \boldsymbol{x}, D = d)\big] dy - \int_{-\infty}^0 \mathbb{P}(Y \leq y \mid R = 0, \mathbf{X} = \boldsymbol{x}, D = d) dy \\
&\quad \leq \int_0^\infty \big[1 - \zeta^{-1} \mathbb{P}(Y \leq y \mid R = 1, \mathbf{X} = \boldsymbol{x}, D = d)\big] dy - \int_{-\infty}^0 \zeta^{-1} \mathbb{P}(Y \leq y \mid R = 1, \mathbf{X} = \boldsymbol{x}, D = d) dy
\end{aligned}$$

where the first equality follows by Lemma 1. I then use the relationship in Display (4) to upper bound the distribution of the outcome for missing observations.

[1] Note that the bound is loose for higher quantiles: for any distributions, there exists $y_\zeta = Q_Y(1/\zeta)$ such that for all $y \geq y_\zeta$ the upper bound is greater than 1. Although this does not pose an issue in deriving the bound, it is certainly possible that the bound can be tighter at the cost of simplicity. For example, we can replace the upper bound in the above expression with $\min\{\zeta \mathbb{P}(Y \leq y \mid R = 1, \mathbf{X} = \boldsymbol{x}, D = d), 1\}$ which guarantees that the upper bound is also a proper distribution function.

Using a similar approach, I can obtain the lower bound as well. Proposition 1 shows the expression of the upper and lower bound of the ATE given the value of the sensitivity parameter ζ.

Proposition 1 (Bound on ATE)**.** *Suppose that the value of $\zeta \geq 1$ is given, and the distribution under the missingness satisfies the inequality in Equation (4). Then, we have the bound on the average treatment effect as $\tau_{LB}(\zeta) \leq \tau \leq \tau_{UB}(\zeta)$ where*

$$\tau_{UB}(\zeta) = \int_{\mathcal{Y}} \Big\{ \mathbb{F}_{w_0(\zeta)}(Y \leq y) - \mathbb{F}_{w_1(1/\zeta)}(Y \leq y) \Big\} dy$$
$$\tau_{LB}(\zeta) = \int_{\mathcal{Y}} \Big\{ \mathbb{F}_{w_0(1/\zeta)}(Y \leq y) - \mathbb{F}_{w_1(\zeta)}(Y \leq y) \Big\} dy$$

where $\mathbb{F}_w(Y \leq y)$ is the weighted cumulative function,

$$\mathbb{F}_{w_d(\zeta)}(Y \leq y) \equiv \mathbb{E}\Bigg\{ \underbrace{\frac{R}{e_d(\boldsymbol{x})/\{e_d(\boldsymbol{x}) + (1 - e_d(\boldsymbol{x}))\zeta\}}}_{\equiv w_d(\zeta)} \mathbf{1}\{Y \leq y\} \;\Bigg|\; D = d \Bigg\}. \tag{5}$$

Proof. See Appendix A.3. □

Proposition 1 shows that ATE can be bounded as a function the weighted cumulative function where the weight depends on the attrition score $e_d(\mathbf{X}_i)$ and the sensitivity parameter ζ. This implies that researchers can assess the validity of results based on the ignorability assumption by varying the value of ζ, while they only need to estimate the attrition score once in their analysis.

Similarly, we obtain the bound on QTE as a function of ζ. The key idea for bounding QTE is that the bound on the quantile function is directly available from the bound on the CDF by taking its left-inverse (see Lemma 2 in Appendix). Then, we obtain the bound on the quantile function for each value of ζ, which enables us to bound QTE for each quantile value $\alpha \in [0, 1]$. Proposition 2 presents the result.

Proposition 2 (Bound on QTE)**.** *Suppose that the value of $\zeta \geq 1$ is given, and the distribution under the missingness satisfies the inequality in Equation (4). Then, the pointwise bound on QTE is given as $\psi_{LB}(\alpha; \zeta) \leq \psi(\alpha) \leq \psi_{UB}(\alpha; \zeta)$ where*

$$\psi_{UB}(\alpha; \zeta) = \mathbb{F}^{\leftarrow}_{w_1(1/\zeta)}(\alpha) - \mathbb{F}^{\leftarrow}_{w_0(\zeta)}(\alpha)$$
$$\psi_{LB}(\alpha; \zeta) = \mathbb{F}^{\leftarrow}_{w_1(\zeta)}(\alpha) - \mathbb{F}^{\leftarrow}_{w_0(1/\zeta)}(\alpha)$$

where $\mathbb{F}^{\leftarrow}$ is a left-inverse of $\mathbb{F}$ in Equation (5).

Proof. See Appendix A.4. □

One attractive feature of the proposed approach is that the sensitivity parameter is shared across different causal estimands. The value of ζ used to bound ATE has the same interpretation of ζ used to

bound QTEs. This feature is attractive because scholars can assess the validity of the choice of sensitivity without referring to specific estimands.

2.3 Sensitivity-Aware Confidence Interval

Given the bounds on ATE and QTE, I now construct that confidence interval that incorporate the information of ζ. A researcher can report the confidence intervals by varying the choice of ζ to show the robustness of the estimate.

First, I discuss how to empirically evaluate the bound given the data. To estimate the bound, we need to estimate the cumulative function,

$$\widehat{\mathbb{F}}_{w_d(\zeta)}(y) = \frac{1}{n_d}\sum_{i=1}^{n} \widehat{w}_{di}(\zeta)\mathbf{1}\{Y_i \leq y\}, \quad \widehat{w}_{di}(\zeta) = R_i\mathbf{1}\{D_i = d\}\left\{1 + \zeta\frac{1-\widehat{e}_d(\mathbf{X})}{\widehat{e}_d(\mathbf{X}_i)}\right\}$$

where we replace the weights with its estimate $\widehat{e}_d(\mathbf{X}_i)$. The weight is computed by plugging in the estimated attrition score $\widehat{e}_d(\mathbf{X}_i)$. Standard methods to estimate the propensity score in the causal inference literature, such as the logistic regression or the covariate balancing propensity score method (Imai and Ratkovic, 2014), can be used to estimate the attrition score.

Then, we obtain the estimator for the upper (lower) bound of ATE by numerically integrating the estimated cumulative function,

$$\widehat{\tau}_{\mathtt{UB}}(\zeta) = \int_{y_-}^{y_+} \left\{\widehat{\mathbb{F}}_{w_0(\zeta)}(y) - \widehat{\mathbb{F}}_{w_1(1/\zeta)}(y)\right\}dy$$

where $y_+ = \max(Y_i)$ and $y_- = \min(Y_i)$. The estimator of QTE is given similarly by numerically inverting the function $\widehat{F}_{w_d(\zeta)}$.

Given the estimator of the bounds, we can now construct a confidence interval as a function of the estimator. Specifically, I construct a confidence interval that covers the *target parameter* with probability $1-\alpha$ (Imbens and Manski, 2004; Vansteelandt et al., 2006). This type of confidence intervals is generally shorter than a confidence interval that has the coverage guarantees on the *bounds* on ATE or QTE (i.e., the confidence interval covers the identification bound with probability $1-\alpha$). In addition to the ease of interpretation, the former type of confidence interval is attractive because when $\zeta = 1$ (i.e., the ignorability assumption holds) the interval naturally collapses to the regular confidence interval on the point estimate.

Given the critical vaue $c_{1-\alpha}$, the $100(1-\alpha)\%$ confidence interval is constructed as

$$\widehat{C}_{1-\alpha}(\tau;\zeta) = \left[\widehat{\tau}_{\mathtt{LB}}(\zeta) - c_{1-\alpha}\sqrt{\widehat{\mathbb{V}}(\widehat{\tau}_{\mathtt{LB}}(\zeta))},\ \widehat{\tau}_{\mathtt{UB}}(\zeta) + c_{1-\alpha}\sqrt{\widehat{\mathbb{V}}(\widehat{\tau}_{\mathtt{UB}}(\zeta))}\right]$$

The variance estimate $\widehat{\mathbb{V}}(\widehat{\tau}_j)$ for $j \in \{\mathtt{LB}, \mathtt{UB}\}$ is obtained via the bootstrap. The validity of bootstrap of this type is shown, for example, in Chernozhukov, Fernández-Val and Melly (2013). I follow Imbens and

Manski (2004) to compute $c_{1-\alpha}$ by numerically solving the following with a grid search,

$$\Phi\left(c_{1-\alpha} + \sqrt{n}\{\widehat{\tau}_{\mathsf{UB}} - \widehat{\tau}_{\mathsf{LB}}\}/\sqrt{\max\left\{\widehat{\mathsf{V}}(\widehat{\tau}_{\mathsf{LB}}), \widehat{\mathsf{V}}(\widehat{\tau}_{\mathsf{UB}})\right\}}\right) - \Phi(-c_{1-\alpha}) = 1-\alpha.$$

3 Empirical Applications

In this section, I apply the proposed method to randomized experiments that have missing outcomes. In the first application, I revisit a randomized experiment with multiple waves of follow-ups, and the outcome suffers from missingness due to attrition. In the second application, I analyze the data from an experiment where the outcome is treated as missing due to "don't know" responses. In both applications, I assess the robustness of the results based on the ignorability assumption by constructing sensitive-aware confidence intervals by changing the value of the sensitivity parameter.

3.1 Randomized Experiment with Differential Attrition

Margalit and Shayo (2021) study what effect interaction with the stock market have on people's socio-economic perception and political preferences. The original authors conduct a randomized experiment where participants receive money to invest in stock market. For simplicity, I focus on two treatment arms in the original study: a group that invests in real stocks (treatment group), and a group that do not invest in stocks at all (control group). The outcome of interest is an index of socioeconomic values (SEV), which is constructed based on survey questions on personal responsibility, economic fairness, inequality, and redistribution.

Multiple waves of survey are conducted to measure the outcome as well as other covariates. Table 1 presents the response rates for the two waves of the survey: Follow-up 1 is a wave conducted immediately after the experiment is completed (short-term outcome), and follow-up 2 is a wave conducted twelve months after the completion of the experiment (long-term outcome), while the baseline corresponds to the beginning of the experiment. The table shows the number of observations in each treatment condition (rows) at time of measurement (columns). Numbers in the parentheses show the response rate relative to the baseline observations. The table suggests that the treatment had a differential impact on the response

Table 1: Response rate of experimental subjects over two waves. The numbers correspond to the number of respondent in each wave of the survey, while the numbers in parenthesis shows the response rate compared with the baseline.

	Baseline	Follow-up 1	Follow-up 2
Treatment	1560	1317 (0.84)	961 (0.62)
Control	520	402 (0.77)	285 (0.55)

rate. Testing against the sharp null of no differential attrition whatsoever (i.e., $R_i(1) = R_i(0)$ for all i) at each wave, I find that the permutation test rejects the null with p-value of 0 for Follow-up 1 and 0.003

for Follow-up 2 (see Appendix C.1.1 for details). This suggests that that control group has significantly higher attrition rate.

To address the possible violation of the assumption of ignorable missingness, I estimate ATE and QTE with varying choices of the sensitivity parameter ζ. To compute the attrition score $e_d(\mathbf{X}_i)$, I use the covariate balancing propensity score method (Imai and Ratkovic, 2014) where the baseline covariates such as age, gender or the pre-treatment outcome are used to construct the score (see Appendix C.1.2 for details).

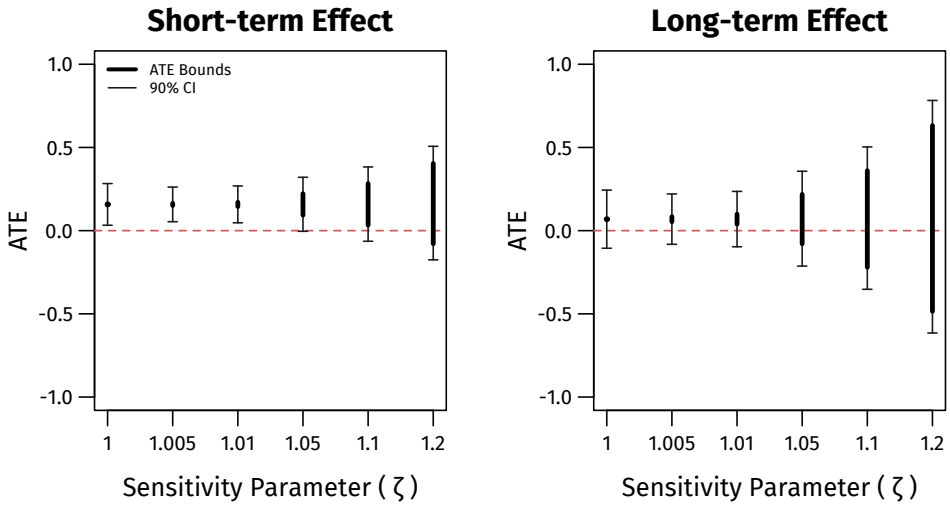

Figure 1: Sensitivity bounds and 90% confidence intervals on average treatment effects (ATE) with different choices of sensitivity parameter, $\zeta \in \{1, 1.005, 1.01, 1.05, 1.1, 1.2\}$. Estimates with $\zeta = 1$ correspond to estimate under conditional independence assumption (Assumption 2).

Figure 1 shows the estimated ATE for the short-term outcome (left panel) and for the long-term outcome (right panel) by different choices of the sensitivity parameters ζ on the x-axis. Thick lines show the estimated identification bound, and thin lines show the 90% confidence intervals. When $\zeta = 1$, we obtain the point estimate with the regular confidence intervals under the ignorability assumption, while for any $\zeta > 1$, the confidence intervals account for the possible violation of the ignorability assumption.

The figure suggests that the effect is robust to the violation of the assumption for the short-term outcome. It shows that even when $\zeta \approx 1.05$, the 90% confidence interval does not cover zero, which implies that the median value among the observed should be between 48th and 53rd percentile among the unobserved. For effect for the long-term outcome on the other hand is less robust because the identification region covers zero at smaller values of ζ; in addition, the effects are statistically distinguishable from zero at the 10% level.

Since the outcome is continuously distributed, scholars can also investigate the quantile treatment effect where the interest is in understanding how the outcome distribution shifts by the treatment. Figure 2 shows the estimated 90% CI on QTE with different choices of the sensitivity parameter ζ. Points

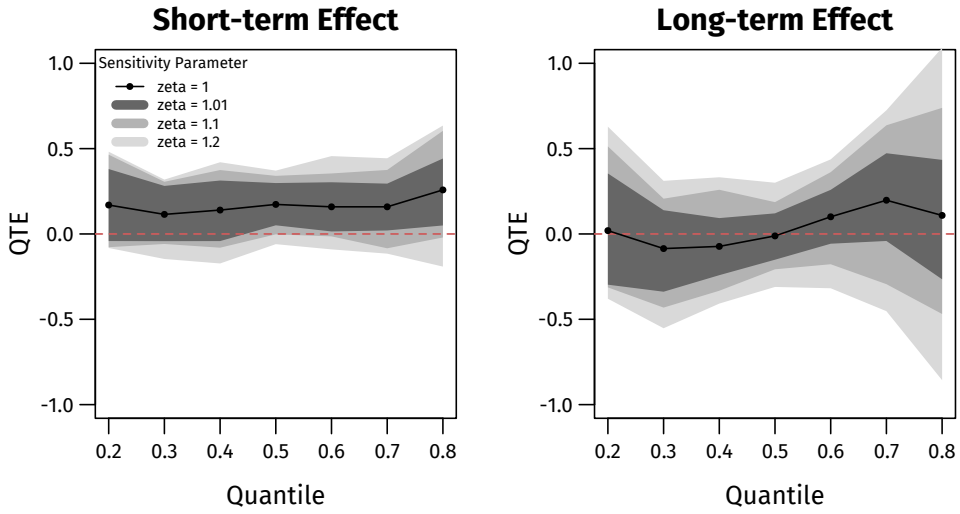

Figure 2: 90% pointwise confidence intervals on the quantile treatment effects (QTE) with different choices of the sensitivity parameter, $\zeta \in \{1.01, 1.1, 1.2\}$. Point estimates are shown for $\zeta = 1$. Estimates with $\zeta = 1$ correspond to estimate under conditional independence assumption (Assumption 2).

represent QTEs evaluated at quantiles from the 20 percentile to the 80 percentile. The figure again shows the robustness of the result for the short-term outcome, but the result is more sensitive to the violation of the assumption: QTEs evaluated above the median are all positive and remain statistically significantly at $\zeta = 1.01$ (thick gray band). On the other hand, the signs of the QTEs for the long-term outcome are not consistent and the 90% confidence interval covers zero for all quantiles evaluated in the figure.

3.2 Randomized Experiment with Missing Outcome due to "Don't Know"

Bisgaard and Slothuus (2018) study how providing a party cue changes voter's attitude toward policy issues. In their experimental study, the authors randomly assign one of the two scripts to respondents. In one script, subjects are told that the government thinks the budget deficit is a serious problem, while in the other script subject are told that the government does not think the budget deficit as a serious problem. After reading the statement, subjects are prompted to rate if they think the issue is a problem on a 5-point scale from "no problem at all" (0) to "a very big problem" (4). The same experiment is conducted on the unemployment issue as well. Approximately, 13% and 3% of experimental subjects chose the DK option for the budget and unemployment issue, respectively.

The original authors conduct a complete case analysis without any covariate adjustments except for a subgroup analysis based on party affiliation. Although the missing rate is not significantly different between the treatment conditions, gender and the level of education are correlated with the response indicator. In addition, the dataset do not include much covariates about the subjects to adjust for, which suggests that analyses under the ignorability assumption might be sensitive to unobserved confounders.

I estimate the attrition score $e_d(\mathbf{X}_i)$ using CBPS with gender, the level of education and age as the

baseline covariates. As in the previous application, I choose the range of the sensitivity parameter as $\zeta \in \{1, 1.005, 1.01, 1.05, 1.1, 1.2\}$. Recall that when $\zeta = 1$, the bound collapses to a point estimate that we would obtain under the ignorability assumption.

The outcome has a discrete support. The proposed method can accommodate the discrete outcome. Let $Y_i \in \{0, \ldots, 4\}$ denote the outcome. Then, the estimate of the bound on ATE is given by replacing integral with sum:

$$\tau \geq \sum_{j=0}^{4} \left\{ \widehat{\mathbb{F}}_{w_0(1/\zeta)}(Y_i \leq j) - \widehat{\mathbb{F}}_{w_1(\zeta)}(Y_i \leq j) \right\}$$
$$\tau \leq \sum_{j=0}^{4} \left\{ \widehat{\mathbb{F}}_{w_0(\zeta)}(Y_i \leq j) - \widehat{\mathbb{F}}_{w_1(1/\zeta)}(Y_i \leq j) \right\}$$

where the cumulative function $\mathbb{F}$ is estimated as

$$\widehat{\mathbb{F}}_{w_d(\zeta)}(Y_i \leq j) = \frac{1}{n_d} \sum_{i=1}^{n} \widehat{w}_d(\zeta) \mathbf{1}\{D_i = d\} \mathbf{1}\{Y_i \leq j\}.$$

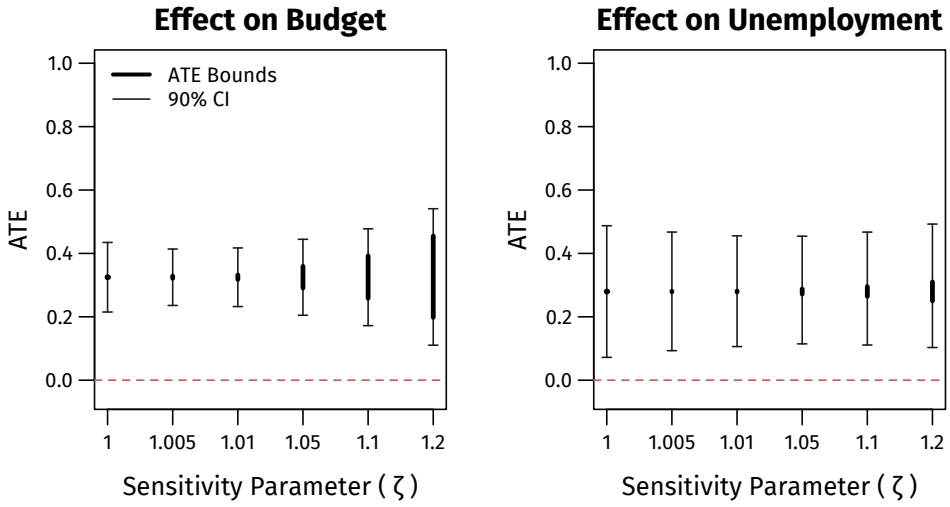

Figure 3: Sensitivity bounds and 90% confidence intervals on average treatment effects (ATE) with different choices of sensitivity parameter, $\zeta \in \{1, 1.005, 1.01, 1.05, 1.1, 1.2\}$. Estimates with $\zeta = 1$ correspond to estimate under conditional independence assumption (Assumption 2).

Figure 3 shows the estimated bounds on ATE and sensitivity-aware confidence intervals. The left (right) panel shows the effect on the budget (unemployment) issue, where the x-axis corresponds to the values of the sensitivity parameter. The figure on the left shows that as the value of ζ increases, the width of the bound (and the confidence interval) increases, while such pattern is less pronounced on the right panel. In both cases, the figure show that the result is quite robust, contrary to what we have found in the

previous application. Even when $\zeta = 1.2$ where we posit that the median value for the missing group is between 40-th and 60-th percentile, the 90% sensitive-aware confidence interval does not cover zero. The result implies the robustness of the original findings.

4 Concluding Remarks

Missing outcome in randomized experiments is a prevalent issue in empirical studies, and complete-case analysis or inverse probability weighting approach are commonly used to address the issue. However, these approaches impose a strong independence (possibly conditional on pre-treatment covariates) assumption, violation of which leads to a biased estimate and inference. In this paper, I have proposed a method to conduct a sensitivity analysis and construct a confidence interval that accounts for the sensitivity. Specifically, the method bounds the ratio of quantiles between the distribution of observed and missing outcomes. I show that the framework is general in that it allows us to bound treatment effects beyond the conventional average treatment effect; in particular I derived a bound on the quantile treatment effects.

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Online Appendix

A Proofs

A.1 Lemmas

Lemma 2. *Let $F(x) = \mathbb{P}(X \leq x)$ denote the CDF of a random variable X. Suppose that $F(x)$ is bounded as $F_-(x) \leq F(x) \leq F_+(x)$ for all $x \in \mathcal{X}$. Then, for $F^{\leftarrow}(\alpha) = \inf\{x\colon F(x) \geq \alpha\}$ we have*

$$F_+^{\leftarrow}(\alpha) \leq F^{\leftarrow}(\alpha) \leq F_-^{\leftarrow}(\alpha)$$

for all $\alpha \in [0,1]$.

A.2 Proof of Result 1

Proof. We have

$$\begin{aligned}
\Pr(Y(d) \leq y) &= \mathbb{E}[\mathbf{1}\{Y(d) \leq y\}] \\
&= \mathbb{E}\big\{\mathbb{E}[\mathbf{1}\{Y(d) \leq y\} \mid \mathbf{X}]\big\} \\
&= \mathbb{E}\big\{\mathbb{E}[\mathbf{1}\{Y(d) \leq y\} \mid \mathbf{X}, D = d, R = 1]\big\} \\
&= \int_{\mathcal{X}} \Pr(Y \leq y \mid \mathbf{X}, D = d, R = 1) dF_{\mathbf{X}}(\boldsymbol{x})
\end{aligned}$$

where the second equality is an application of the law of iterated expectation, the third equality is due to Assumption 1 and 2, and the last line follows by using the relationship $Y^{obs} = R\{DY(1) + (1 - D)Y(0)\}$. □

A.3 Proof of Proposition 1

Proof. Without loss of generality, I assume that $\mathcal{Y} = (-\infty, \infty)$, and derive the formula for the upper bound $\tau_{\mathtt{UB}}(\zeta)$. The lower bound can be derived similarly.

I first derive the upper and lower bound of the conditional expectation of the outcome for the missing observations. Using the bounded ratio of two CDFs and Lemma 1, we have

$$\begin{aligned}
&\mathbb{E}[Y \mid R = 0, \mathbf{X} = \boldsymbol{x}, D = d] \\
&\quad = \int_0^\infty \big[1 - \mathbb{P}(Y \leq y \mid R = 0, \mathbf{X} = \boldsymbol{x}, D = d)\big] dy - \int_{-\infty}^0 \mathbb{P}(Y \leq y \mid R = 0, \mathbf{X} = \boldsymbol{x}, D = d) dy \\
&\quad \leq \int_0^\infty \big[1 - \zeta^{-1}\mathbb{P}(Y \leq y \mid R = 1, \mathbf{X} = \boldsymbol{x}, D = d)\big] dy - \int_{-\infty}^0 \zeta^{-1}\mathbb{P}(Y \leq y \mid R = 1, \mathbf{X} = \boldsymbol{x}, D = d) dy
\end{aligned}$$

and

$$
\begin{aligned}
&\mathbb{E}[Y \mid R=0, \mathbf{X}=\boldsymbol{x}, D=d] \\
&\quad = \int_0^\infty \big[1 - \mathbb{P}(Y \le y \mid R=0, \mathbf{X}=\boldsymbol{x}, D=d)\big]dy - \int_{-\infty}^0 \mathbb{P}(Y \le y \mid R=0, \mathbf{X}=\boldsymbol{x}, D=d)dy \\
&\quad \ge \int_0^\infty \big[1 - \zeta\mathbb{P}(Y \le y \mid R=1, \mathbf{X}=\boldsymbol{x}, D=d)\big]dy - \int_{-\infty}^0 \zeta\mathbb{P}(Y \le y \mid R=1, \mathbf{X}=\boldsymbol{x}, D=d)dy
\end{aligned}
$$

Applying the above bounds on the conditional expectation, we have that

$$
\begin{aligned}
&\int_{\mathcal{X}} \mathbb{E}[Y \mid R=0, \mathbf{X}=\boldsymbol{x}, D=d] \Pr(R=0 \mid \mathbf{X}=\boldsymbol{x}, D=d) dF(\boldsymbol{x}) \\
&\qquad \le \int_{\mathcal{X}} \left\{ \int_0^\infty \left[1 - \mathbb{E}\left\{ \frac{R\mathbf{1}\{Y \le y\}}{e_d(\boldsymbol{x})} \;\middle|\; \mathbf{X}=\boldsymbol{x}, D=d \right\} \cdot \frac{1-e_d(\boldsymbol{x})}{\zeta} \right] dy \right. \\
&\qquad\qquad \left. - \int_{-\infty}^0 \mathbb{E}\left[\frac{R\mathbf{1}\{Y \le y\}}{e_d(\boldsymbol{x})} \;\middle|\; \mathbf{X}=\boldsymbol{x}, D=d \right] dy \cdot \frac{1-e_d(\boldsymbol{x})}{\zeta} \right\} f(\boldsymbol{x}) d\boldsymbol{x}
\end{aligned}
$$

and

$$
\begin{aligned}
&\int_{\mathcal{X}} \mathbb{E}[Y \mid R=1, \mathbf{X}=\boldsymbol{x}, D=d] \Pr(R=1 \mid \mathbf{X}=\boldsymbol{x}, D=d) dF(\boldsymbol{x}) \\
&\qquad = \int_{\mathcal{X}} \left\{ \int_0^\infty \left[1 - \mathbb{E}\left\{ \frac{R\mathbf{1}\{Y \le y\}}{e_d(\boldsymbol{x})} \;\middle|\; \mathbf{X}=\boldsymbol{x}, D=d \right\} \cdot e_d(\boldsymbol{x}) \right] dy \right. \\
&\qquad\qquad \left. - \int_{-\infty}^0 \mathbb{E}\left[\frac{R\mathbf{1}\{Y \le y\}}{e_d(\boldsymbol{x})} \;\middle|\; \mathbf{X}=\boldsymbol{x}, D=d \right] dy \cdot e_d(\boldsymbol{x}) \right\} f(\boldsymbol{x}) d\boldsymbol{x}.
\end{aligned}
$$

where I used the following equality

$$
\mathbb{P}(Y \le y \mid R=1, \mathbf{X}=\boldsymbol{x}, D=d) = \mathbb{E}\left\{ \frac{R\mathbf{1}\{Y \le y\}}{e_d(\boldsymbol{x})} \;\middle|\; \mathbf{X}=\boldsymbol{x}, D=d \right\}.
$$

with $e_d(\mathbf{X}_i) = \Pr(R_i = 1 \mid \mathbf{X}_i, D=d)$.

Then, the upper bound of $\mathbb{E}[Y \mid D=1]$ is given by

$$
\begin{aligned}
\mathbb{E}[Y \mid D=1] &= \int_{\mathcal{X}} \Big\{ \mathbb{E}[Y \mid R=1, D=1, \mathbf{X}=\boldsymbol{x}] e_1(\boldsymbol{x}) + \mathbb{E}[Y \mid R=0, D=1, \mathbf{X}=\boldsymbol{x}](1-e_1(\boldsymbol{x})) \Big\} dF(\boldsymbol{x}) \\
&\le \int_{\mathcal{X}} \left\{ \int_0^\infty \left[1 - \mathbb{E}\left\{ \frac{R\mathbf{1}\{Y \le y\}}{e_1(\boldsymbol{x})/\{e_1(\boldsymbol{x}) + (1-e_1(\boldsymbol{x}))/\zeta\}} \;\middle|\; \mathbf{X}=\boldsymbol{x}, D=1 \right\} \right] dy \right. \\
&\qquad \left. - \int_{-\infty}^0 \mathbb{E}\left[\frac{R\mathbf{1}\{Y \le y\}}{e_1(\boldsymbol{x})/\{e_1(\boldsymbol{x}) + (1-e_1(\boldsymbol{x}))/\zeta\}} \;\middle|\; \mathbf{X}=\boldsymbol{x}, D=1 \right] dy \right\} dF(\boldsymbol{x}) \\
&= \int_0^\infty \left[1 - \mathbb{E}\left\{ \frac{R\mathbf{1}\{Y \le y\}}{e_1(\boldsymbol{x})/\{e_1(\boldsymbol{x}) + (1-e_1(\boldsymbol{x}))/\zeta\}} \;\middle|\; D=1 \right\} \right] dy \\
&\qquad - \int_{-\infty}^0 \mathbb{E}\left[\frac{R\mathbf{1}\{Y \le y\}}{e_1(\boldsymbol{x})/\{e_1(\boldsymbol{x}) + (1-e_1(\boldsymbol{x}))/\zeta\}} \;\middle|\; D=1 \right] dy
\end{aligned}
$$

$$\equiv \int_0^\infty \Big\{1 - \mathbb{F}_{w_1(1/\zeta)}(Y \le y)\Big\}dy - \int_{-\infty}^0 \mathbb{F}_{w_1(1/\zeta)}(Y \le y)dy$$

where the first inequality is due to the assumption on the ratio of two CDFs, and the third line follows by exchanging the order of two integrals.

Similarly, we obtain the lower bound of $\mathbb{E}[Y \mid D = 0]$ as

$$\mathbb{E}[Y \mid D = 0] \ge \int_0^\infty \Big\{1 - \mathbb{F}_{w_0(\zeta)}(Y \le y)\Big\}dy - \int_{-\infty}^0 \mathbb{F}_{w_0(\zeta)}(Y \le y)dy$$

Combining the two bounds, we obtain the upper bound of ATE as

$$\begin{aligned}
\tau_{\mathsf{ATE}} &\le \int_0^\infty \Big\{1 - \mathbb{F}_{w_1(1/\zeta)}(Y \le y)\Big\}dy - \int_{-\infty}^0 \mathbb{F}_{w_1(1/\zeta)}(Y \le y)dy \\
&\quad - \left\{ \int_0^\infty \Big\{1 - \mathbb{F}_{w_0(\zeta)}(Y \le y)\Big\}dy - \int_{-\infty}^0 \mathbb{F}_{w_0(\zeta)}(Y \le y)dy \right\} \\
&= \int_{\mathcal{Y}} \Big\{\mathbb{F}_{w_0(\zeta)}(Y \le y) - \mathbb{F}_{w_1(1/\zeta)}(Y \le y)\Big\}dy \\
&\equiv \tau_{\mathsf{UB}}(\zeta).
\end{aligned}$$

which completes the proof. □

A.4 Proof of Proposition 2

Proof. We have

$$\begin{aligned}
\mathbb{P}(Y \le y \mid D = d, \mathbf{X} = \boldsymbol{x}) &= \mathbb{P}(Y \le y \mid \mathbf{X} = \boldsymbol{x}, D = d, R = 1)\Pr(R = 1 \mid \mathbf{X} = \boldsymbol{x}, D = d) \\
&\quad + \mathbb{P}(Y \le y \mid \mathbf{X} = \boldsymbol{x}, D = d, R = 0)\Pr(R = 0 \mid \mathbf{X} = \boldsymbol{x}, D = d) \\
&\le \mathbb{P}(Y \le y \mid \mathbf{X} = \boldsymbol{x}, D = d, R = 1)e_d(\boldsymbol{x}) \\
&\quad + \zeta\mathbb{P}(Y \le y \mid \mathbf{X} = \boldsymbol{x}, D = d, R = 1)\{1 - e_d(\boldsymbol{x})\} \\
&= \mathbb{E}\left\{ \frac{R\mathbf{1}\{Y \le y\}}{e_d(\boldsymbol{x})/\{e_d(\boldsymbol{x}) + (1 - e_d(\boldsymbol{x}))\zeta\}} \,\middle|\, \mathbf{X} = \boldsymbol{x}, D = d \right\}
\end{aligned}$$

and similarly

$$\mathbb{P}(Y \le y \mid D = d, \mathbf{X} = \boldsymbol{x}) \ge \mathbb{E}\left\{ \frac{R\mathbf{1}\{Y \le y\}}{e_d(\boldsymbol{x})/\{e_d(\boldsymbol{x}) + (1 - e_d(\boldsymbol{x}))/\zeta\}} \,\middle|\, \mathbf{X} = \boldsymbol{x}, D = d \right\}$$

Integrating both terms over $\mathbf{X}$, we arrive at the bound on the conditional CDF,

$$\mathbb{F}_{w_d(1/\zeta)}(Y \le y) \le \mathbb{P}(Y \le y \mid D = d) \le \mathbb{F}_{w_d(\zeta)}(Y \le y)$$

where $w_{di}(\zeta) = R_i \mathbf{1}\{D_i = d\}\{1 + \zeta(1 - e_d(\mathbf{X}_i))/e_d(\mathbf{X}_i)\}$.

To obtain the bound on the quantile treatment effect, I invert the cumulative functions. Applying the result in Theorem 1 of Chernozhukov et al. (2020) we have

$$\mathbb{F}^{\leftarrow}_{w_d(\zeta)}(\alpha) \leq F_d^{\leftarrow}(\alpha) \leq \mathbb{F}^{\leftarrow}_{w_d(1/\zeta)}(\alpha)$$

Note that it is possible to have $\mathbb{F}_{w_d(\zeta)}(y) \geq 1$ for some y. The bound on the quantile function is valid as long as the bound is evaluated at $\alpha \in [0, 1]$.

Then, the upper (lower) bound of $\psi(\alpha) = F_1^{\leftarrow}(\alpha) - F_0^{\leftarrow}(\alpha)$ can be obtained by taking the pointwise Minkowski difference between two bounds. □

B Extension to Observational Studies

In the observational studies, the treatment assignment is not randomized. Instead, researchers assume that conditional on a rich set of covariates the treatment is independent of the potential outcome.

Assumption 3 (Section-on-observable)**.** $Y_i(d) \perp\!\!\!\perp D_i \mid \mathbf{X}_i$.

Result 2 (Identification of the distribution of the potential outcome)**.** *Under Assumption 3, the distribution of the potential outcome is identified as*

$$\Pr\left(Y_i(d) \leq y\right) = \mathbb{E}\left\{\frac{R_i \mathbf{1}\{D_i = d\}\mathbf{1}\{Y_i \leq y\}}{e_d(\mathbf{X}_i)\pi(\mathbf{X}_i)^d\{1 - \pi(\mathbf{X}_i)\}^{1-d}}\right\}$$

where $\pi(\mathbf{X}_i) = \Pr(D_i = 1 \mid \mathbf{X}_i)$ *is the propensity score.*

Given the bound on the ratio with ζ, the upper bound on the potential outcome is given by

$$\Pr(Y_i(d) \leq y) \leq \mathbb{E}\left\{\frac{\mathbf{1}\{D_i = d\}}{\pi(\mathbf{X}_i)^d\{1 - \pi(\mathbf{X}_i)\}^{1-d}} \frac{R_i \mathbf{1}\{Y_i \leq y\}}{e_d(\boldsymbol{x})/\{e_d(\boldsymbol{x}) + (1 - e_d(\boldsymbol{x}))\zeta\}}\right\}$$

Therefore, the bound on ATE is given by the expression in Proposition 1 with a different form of the weight

$$w_{di}(\zeta) = \frac{\mathbf{1}\{D_i = d\}}{\pi(\mathbf{X}_i)^d\{1 - \pi(\mathbf{X}_i)\}^{1-d}} \frac{R_i\{e_d(\boldsymbol{x}) + (1 - e_d(\boldsymbol{x}))\zeta\}}{e_d(\boldsymbol{x})}.$$

C Additional Empirical Results

C.1 Margalit and Shayo (2021)

C.1.1 Testing the differential attrition

To test if the treatment has effect on the attrition, I conduct a randomized inference for the differential attrition, where the null hypothesis is no treatment effect on the attrition whatsoever, $R_i(1) = R_i(0)$ for all i. Figure 4 shows the result: The histogram shows the distribution of the test statistic under null, approximated by 2000 Monte Carlo draws of treatment assignments. The red vertical line shows the observed test statistic.

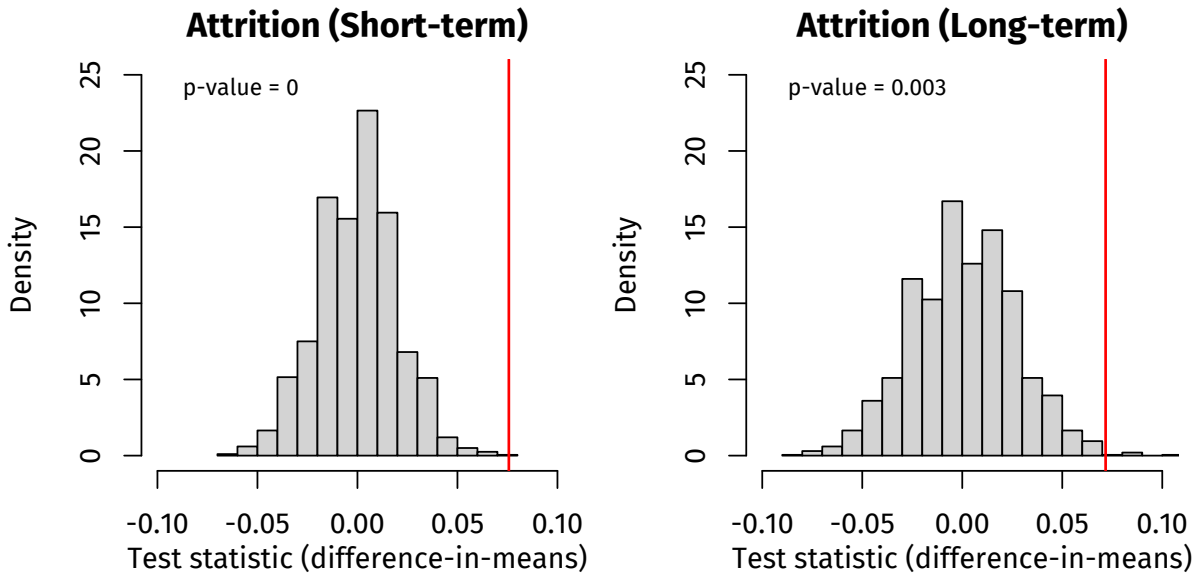

Figure 4: Randomized inference against the sharp null of no attrition whatsoever, $R_i(1) = R_i(0)$ for all i. The vertical line in red indicates the observed test statistic. The histogram shows the distribution of the test statistic under the null, which is generated by the 2000 draws of the treatment assignment.

The p-value for the short-term outcome is 0.000 and for the long-term outcome is 0.0003, and therefore we can reject the null in both cases.

C.1.2 Computing the attrition score

In order to compute the attrition score for the short- and long-term outcome, I use the covariate balancing propensity score method to estimate the probability of attrition given baseline covariates.

Table 2: Estimated coefficients from `CBPS` on the short-term outcome.

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	0.156	0.891	0.175	0.861
age_w1	0.001	0.115	0.007	0.995
invest_w1	-0.098	0.091	-1.072	0.284
risk_1_w1	-0.014	0.102	-0.138	0.890
equality_1_w1	0.013	0.133	0.100	0.920
female_w1	0.129	0.103	1.257	0.209
blame_system_w2	0.118	0.169	0.701	0.483
born_uk_w2	-0.081	0.104	-0.778	0.436
voted_cons_15	-0.055	0.124	-0.447	0.655
voted_lab_15	0.038	0.093	0.405	0.685
voted_ukip_15	0.180	0.119	1.513	0.130
lninc_missing_w2	0.161	0.102	1.573	0.116
SV_pre	0.082	0.204	0.405	0.686
B_risk_1_pre	0.103	0.096	1.073	0.283
B_trust_1_pre	-0.096	0.156	-0.617	0.537
B_equality_1_pre	0.046	0.115	0.399	0.690
B_blame_system_pre	-0.105	0.115	-0.917	0.359
B_bjw_pre	-0.020	0.113	-0.177	0.859
B_luck_pre	0.047	0.105	0.446	0.655
B_religious_w2	-0.015	0.086	-0.179	0.858
fl_big3_w1	0.004	0.095	0.039	0.969
B_jobloss_help_pre	0.262	0.110	2.387	0.017

Table 3: Estimated coefficients from `CBPS` on the long-term outcome.

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	0.156	0.891	0.175	0.861
age_w1	0.001	0.115	0.007	0.995
invest_w1	-0.098	0.091	-1.072	0.284
risk_1_w1	-0.014	0.102	-0.138	0.890
equality_1_w1	0.013	0.133	0.100	0.920
female_w1	0.129	0.103	1.257	0.209
blame_system_w2	0.118	0.169	0.701	0.483
born_uk_w2	-0.081	0.104	-0.778	0.436
voted_cons_15	-0.055	0.124	-0.447	0.655
voted_lab_15	0.038	0.093	0.405	0.685
voted_ukip_15	0.180	0.119	1.513	0.130
lninc_missing_w2	0.161	0.102	1.573	0.116
SV_pre	0.082	0.204	0.405	0.686
B_risk_1_pre	0.103	0.096	1.073	0.283
B_trust_1_pre	-0.096	0.156	-0.617	0.537
B_equality_1_pre	0.046	0.115	0.399	0.690
B_blame_system_pre	-0.105	0.115	-0.917	0.359
B_bjw_pre	-0.020	0.113	-0.177	0.859
B_luck_pre	0.047	0.105	0.446	0.655
B_religious_w2	-0.015	0.086	-0.179	0.858
fl_big3_w1	0.004	0.095	0.039	0.969
B_jobloss_help_pre	0.262	0.110	2.387	0.017