Difference-in-Differences for Ordinal Outcomes

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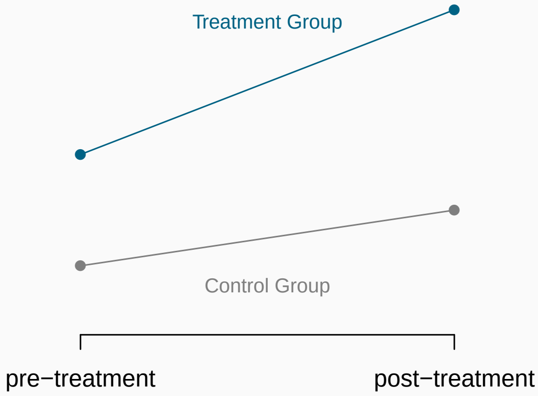

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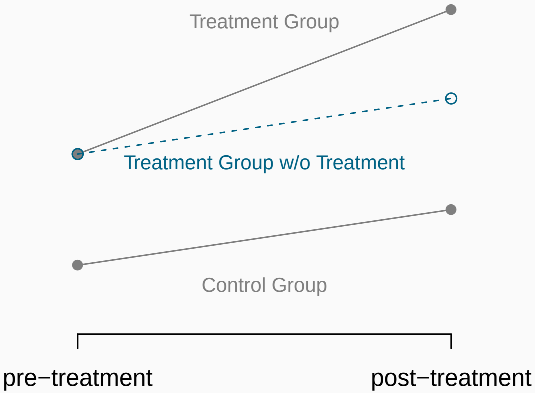

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- Application: Revisit a recent debate on the relationship between the mass shootings and the attitude toward gun control

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Contributions: New identification strategy & diagnostic tool

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  $\rightsquigarrow$ Equivalence based test to assess the plausibility of the assumption

Mass Shootings and Attitudes toward Gun Control

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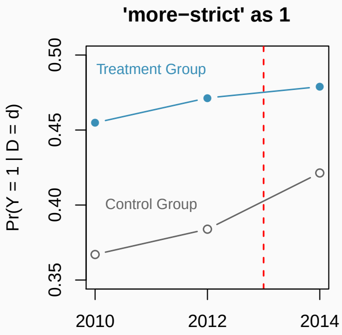

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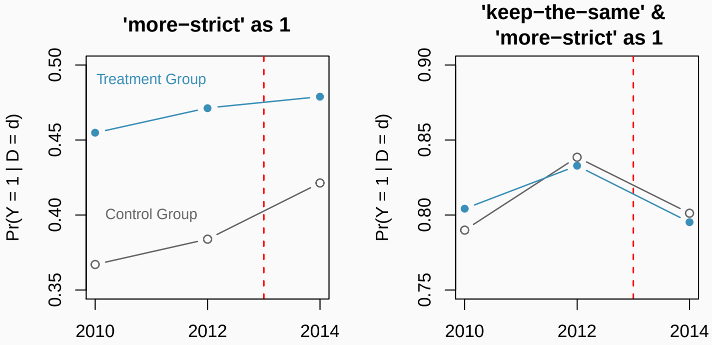

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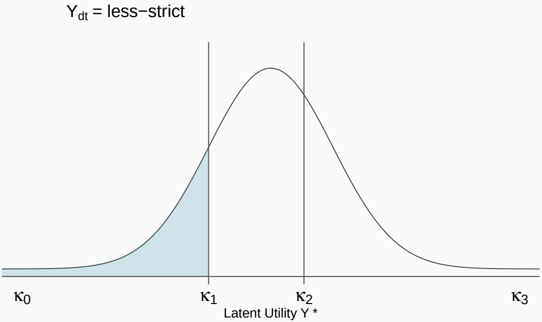

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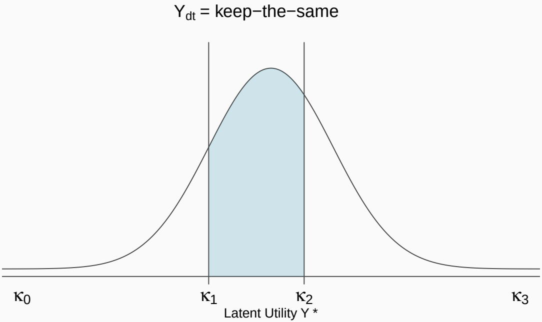

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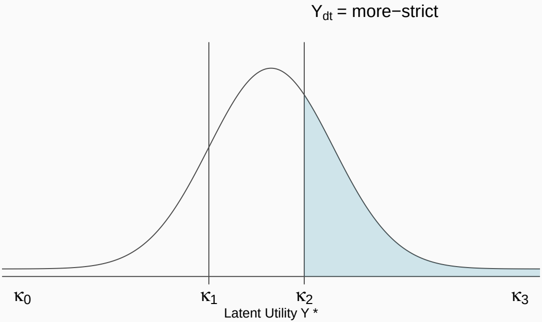

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- Location-scale family: Imposing distribution on Y^*_{dt}

$$Y^*_{dt} \sim \underbrace{\mu_{dt}}_{\text{location}} + \underbrace{\sigma_{dt}}_{\text{scale}} U$$

where U belongs to a parametric family (e.g., normal, logistic, t-dist.)

Main Result

- Distributional parallel-trends assumption (Athey & Imbens 2006)

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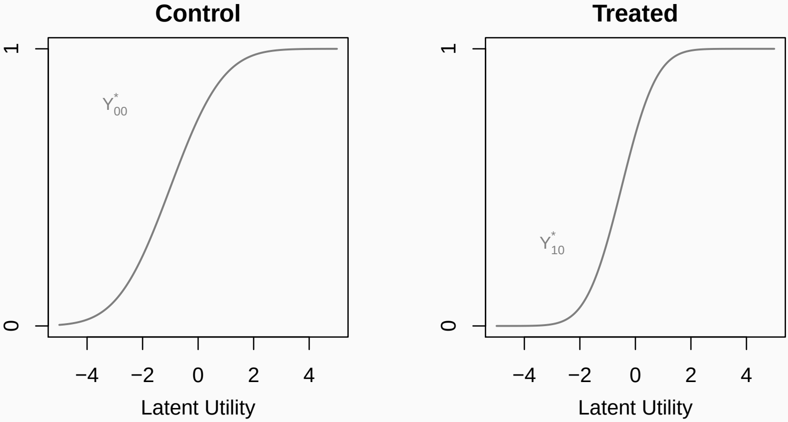

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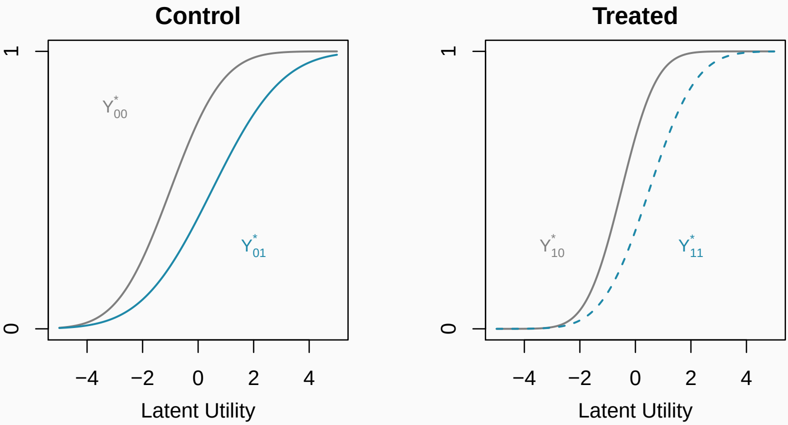

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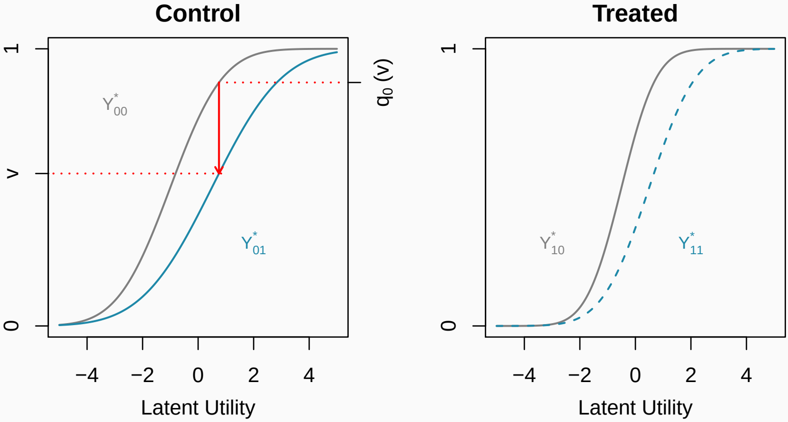

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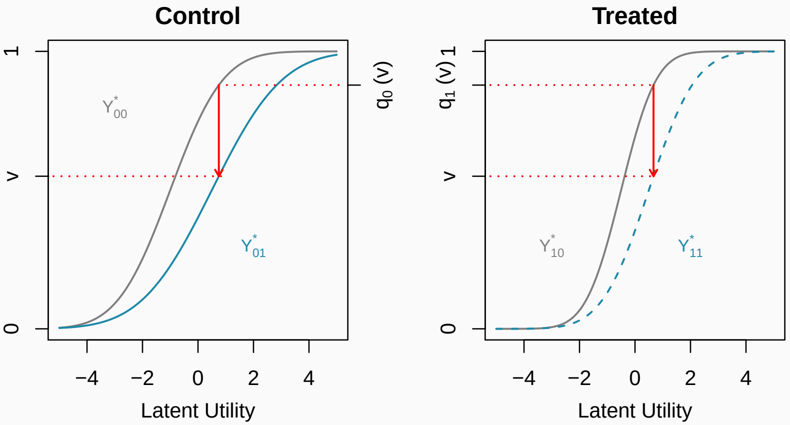

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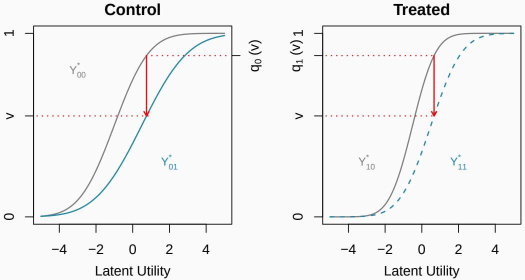

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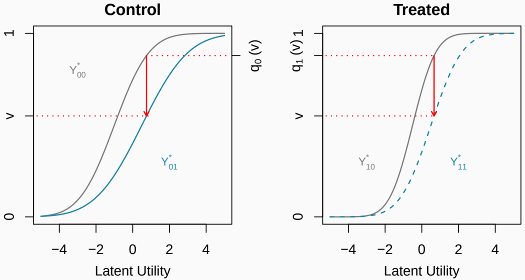

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- When variances are constant $\sigma_{dt} = \sigma$, recovers the usual parallel trends form

$$\mu_{11} - \mu_{10} = \mu_{01} - \mu_{00}$$

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▸ Cutoff ▸ Mean-Variance

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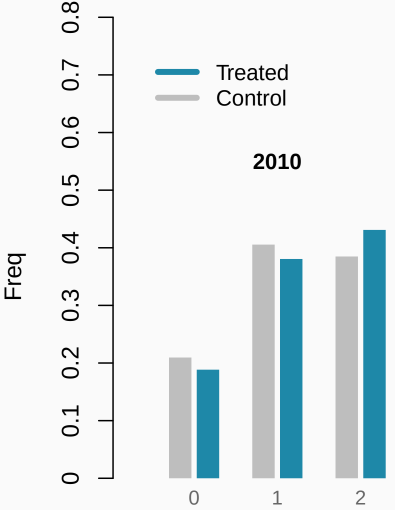

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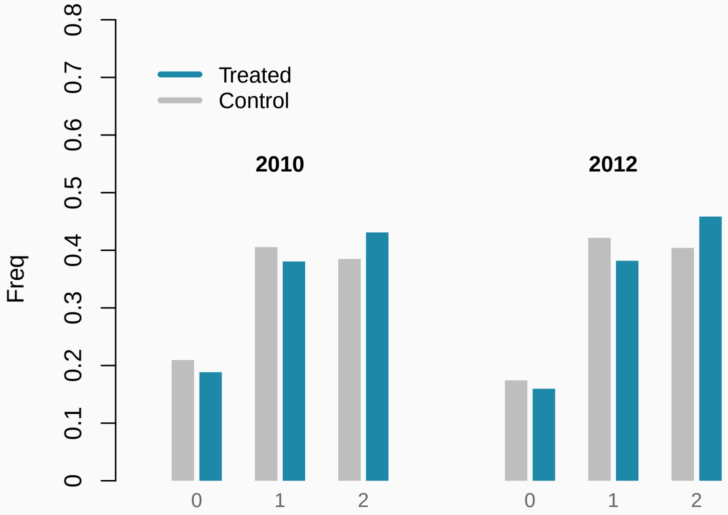

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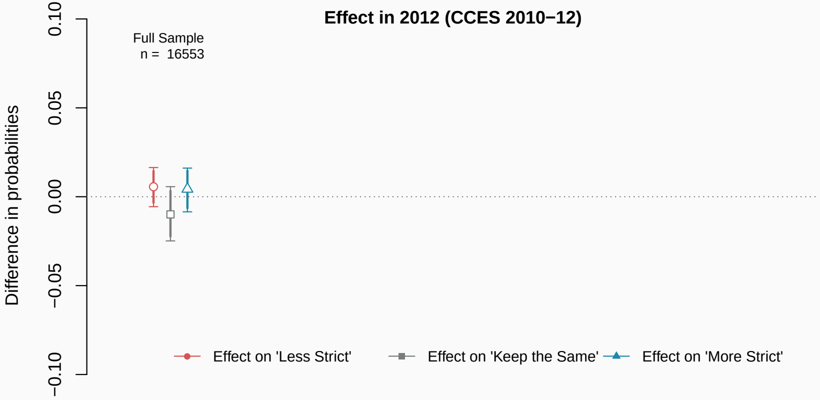

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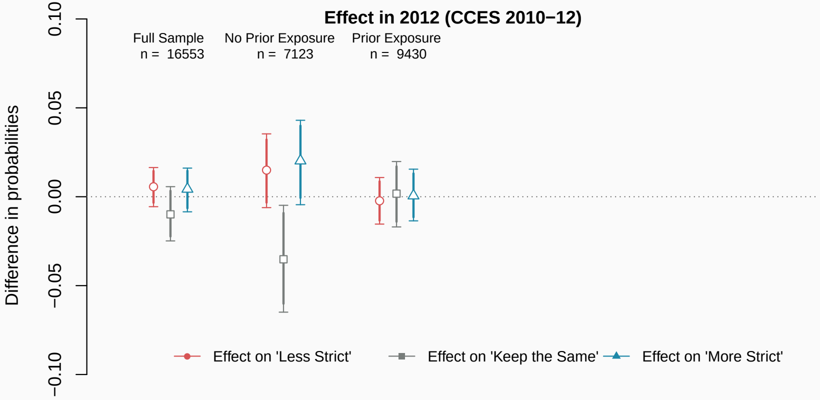

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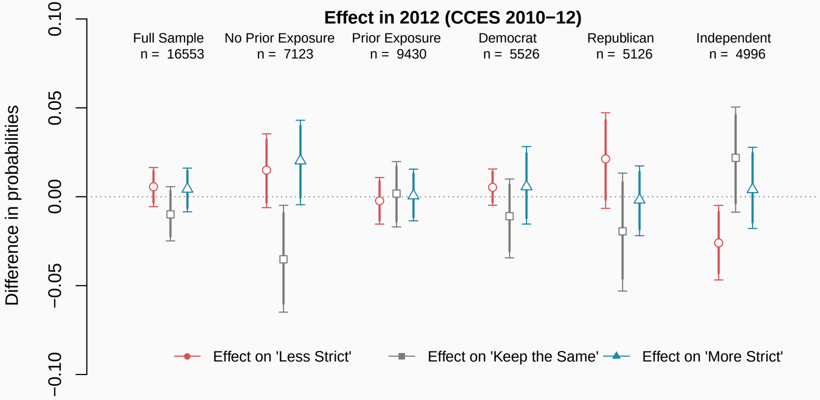

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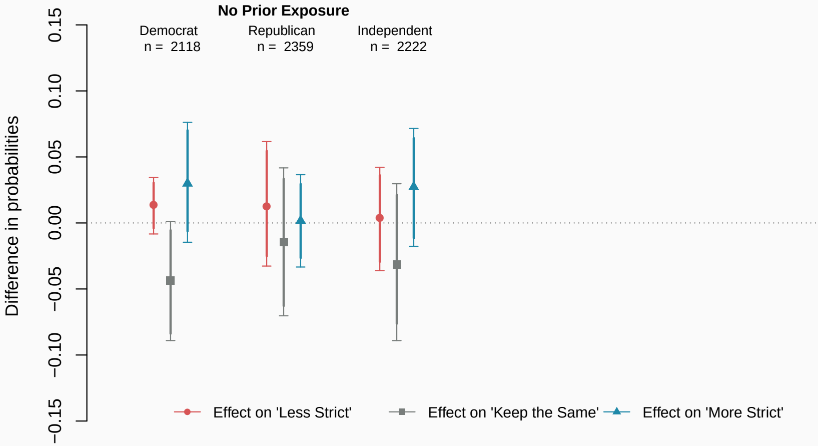

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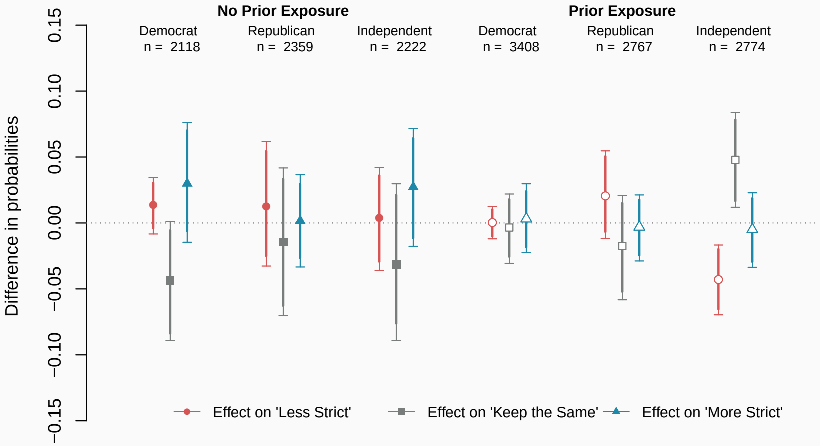

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- Construct one-sided point-wise CIs: $\widehat{U}_{1-\alpha}(v)$ and $\widehat{L}_{1-\alpha}(v)$

$$\text{reject}: H_0^+ \text{ at } \alpha \text{ level} \iff \max_{v\in[0,1]} \widehat{U}_{1-\alpha}(v) < \delta$$

$\rightsquigarrow$ We reject H_0 if we reject both H_0^+ and H_0^-

CI construction | Choose delta

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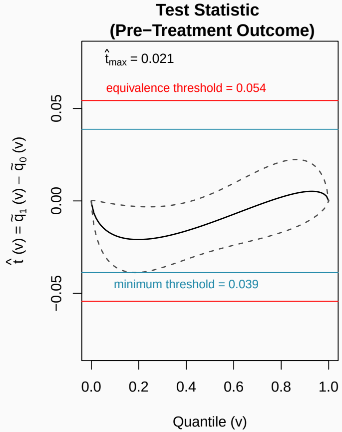

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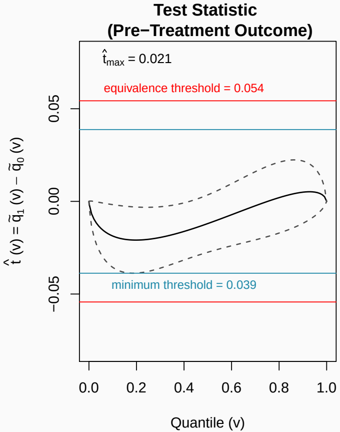

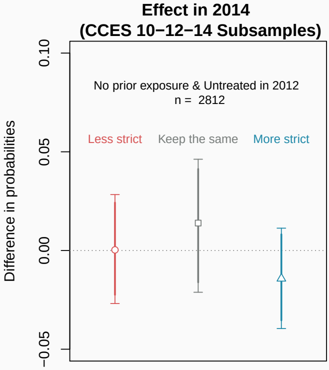

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  - Find that effects are concentrated among Democrats who do not have "prior exposure" to shootings and among Independents

References

- Yamauchi, Soichiro. (2020). "Difference-in-Differences for Ordinal Outcomes: Application to the Effect of Mass Shootings on Attitudes towards Gun Control" *Working Paper*.
- `orddid`: R package for implementing the difference-in-difference for the ordinal outcomes. Available at github.com/soichiroy/orddid

Send comments and suggestions to
syamauchi@g.harvard.edu

For more information
soichiroy.github.io

Additional Results

Treating as Continuous Outcome

- Consider a cross-sectional setting:

$$\zeta_j = \Pr(Y_i(1) = j) - \Pr(Y_i(0) = j)$$

- Rescale the outcome: $\widetilde{Y}_i = Y_i/(J-1)$
- The difference-in-means estiamtor on $\widetilde{Y}_i$ can be written as

$$\widehat{\tau}_{\text{DiM}} = \sum_{j=1}^{J} (J-j)^{-1} \widehat{\zeta}_j$$

  where

$$\widehat{\tau}_{\text{DiM}} = \frac{1}{n_1} \sum_{i=1}^{n} D_i Y_i - \frac{1}{n_0} \sum_{i=1}^{n} (1 - D_i) Y_i$$

  $\rightsquigarrow$ Weighted average of $\widehat{\zeta}_j$ with weights are $1/(J-j)$
- This can potentially cancel out the effects: E.g., $\widehat{\zeta}_1 > 0$ and $\widehat{\zeta}_2 < 0$

Invariance of Causal Effect to Choice of Cutoffs

- **Proposition**: Suppose $Y_{it} \in \mathcal{J} \equiv \{0, 1, 2\}$ and $U \sim \mathcal{N}(0, 1)$. Let $\boldsymbol{\kappa}$ and $\boldsymbol{\kappa}'$ be the different sets of cutoffs. Then, for all $j \in \mathcal{J}$.

$$\widehat{\zeta}_j(\boldsymbol{\kappa}) = \widehat{\zeta}_j(\boldsymbol{\kappa}')$$

- **Intuition**:

(1) Assumption is imposed on the quantile scale (i.e., distributional PT)

$\rightsquigarrow$ counterfactual distribution is identified as long as quantile info. is preserved

(2) Changing cutoffs affect mean & scale $\rightsquigarrow$ transform the latent variables

(3) But quantile information is preserved, $\Pr(Y^* \leq \kappa_1) = \Pr(\tilde{Y}^* \leq \kappa_1')$

$$\int_{\kappa_1}^{\kappa_2} \phi((y^* - \mu_{00})/\sigma_{00}) dy^* = \underbrace{\Pr(Y_{00} = 1)}_{\text{observed prob.}} = \int_{\kappa_1'}^{\kappa_2'} \phi((y^* - \mu_{00}')/\sigma_{00}') dy^*$$

$\rightsquigarrow$ uniquely recovers the counterfactual distribution Y_{11}^*

Back

Identification of Latent Variables

- Suppose that the cutoffs are fixed at κ_1 and κ_2 for $Y_{dt} = j \in \{0, 1, 2\}$. Then, μ_{dt} and σ_{dt} in $Y^*_{dt} \sim \mu_{dt} + \sigma_{dt} U$ are uniquely identified from the observed probability distribution.
- Proof: Suppose that U has the density $f_U(u)$. Then, we can form a non-linear system of equations

$$\Pr(Y_{dt} = 0) = \int_{-\infty}^{\kappa_1} f_U((y^* - \mu_{dt})/\sigma_{dt}) dy^*$$
$$\Pr(Y_{dt} = 2) = \int_{\kappa_2}^{\infty} f_U((y^* - \mu_{dt})/\sigma_{dt}) dy^*$$

which are sufficient for estimating μ and σ.

Back

Alternative Formula of Identification

- Suppose $Y_{dt} = j \in \{0, 1, 2\}$. Let $v_1 = F_{01}(\kappa_1)$ and $v_2 = F_{01}(\kappa_2)$ where $\boldsymbol{\kappa}$ is a set of fixed cutoffs.
- Under the assumptions, we identify μ_{11} and σ_{11} by the following system of non-linear equations:

$$q_0(v_1) = \int_{-\infty}^{F_{10}^{-1}(v_1)} f_U((y^* - \mu_{11})/\sigma_{11}) dy^*$$
$$q_0(v_2) = \int_{-\infty}^{F_{10}^{-1}(v_2)} f_U((y^* - \mu_{11})/\sigma_{11}) dy^*.$$

Constructing Confidence Intervals for Testing

- Let $t(v) = \tilde{q}_1(v) - \tilde{q}_0(v)$
- Point-wise upper and lower $(1-\alpha)$ level confidence intervals:

$$\widehat{U}_{1-\alpha}(v) = \hat{t}(v) + \Phi^{-1}(1-\alpha)\sqrt{\mathrm{Var}(\hat{t}(v))/n}$$
$$\widehat{L}_{1-\alpha}(v) = \hat{t}(v) - \Phi^{-1}(1-\alpha)\sqrt{\mathrm{Var}(\hat{t}(v))/n}$$

- **Proposition**

$$\Pr\left(\max_{v\in[0,1]} t(v) \leq \max_{v'\in[0,1]} \widehat{U}_{1-\alpha}(v')\right) \geq 1-\alpha$$
$$\Pr\left(\min_{v\in[0,1]} t(v) \geq \min_{v'\in[0,1]} \widehat{L}_{1-\alpha}(v')\right) \geq 1-\alpha$$

Back

Choosing Delta

- Value of δ reflect the admissible level of “non equivalence”

  $\rightsquigarrow$ Larger values of δ correspond to lenient thresholds
- Calibrate δ based on the rejection threshold for the KS test

$$\delta_n = \min\left\{\sqrt{-\log(\alpha)/2}\sqrt{\frac{n_1+n_0}{n_1 n_0}}, 1\right\}$$

  and take $\alpha = 0.05$
- Can report the equivalence CI: minimum possible value of δ at α level

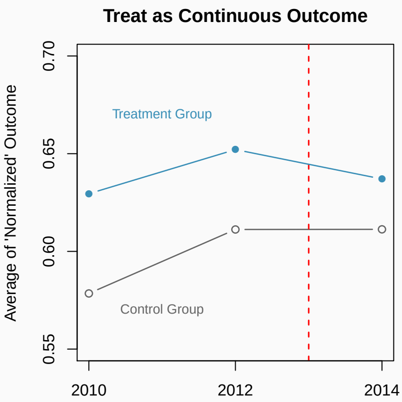
$$\delta_{\min,n} = \max_{v\in[0,1]}\left\{\left|\widehat{U}_{1-\alpha}(v)\right|, \left|\widehat{L}_{1-\alpha}(v)\right|\right\}$$

  $\rightsquigarrow$ Equivalence CI is given by $[-\delta_{\min,n}, \delta_{\min,n}]$

Back

Additional Empirical Analysis

Treating as a Continuous Outcome



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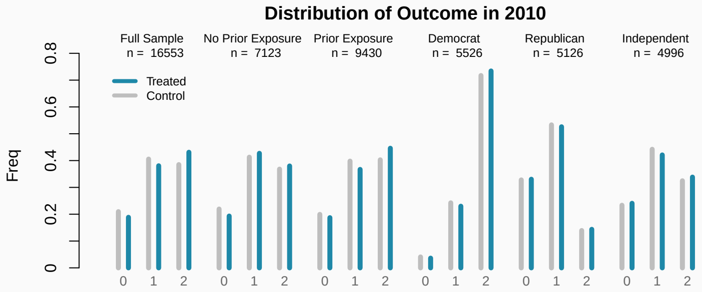
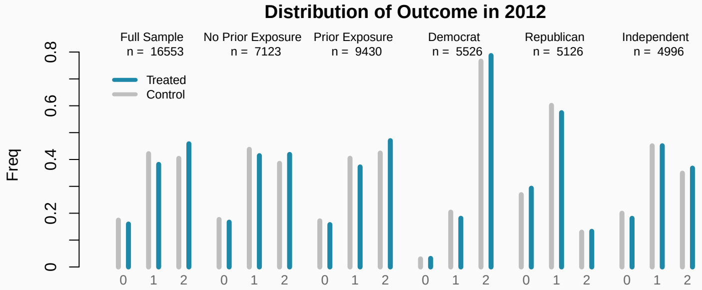
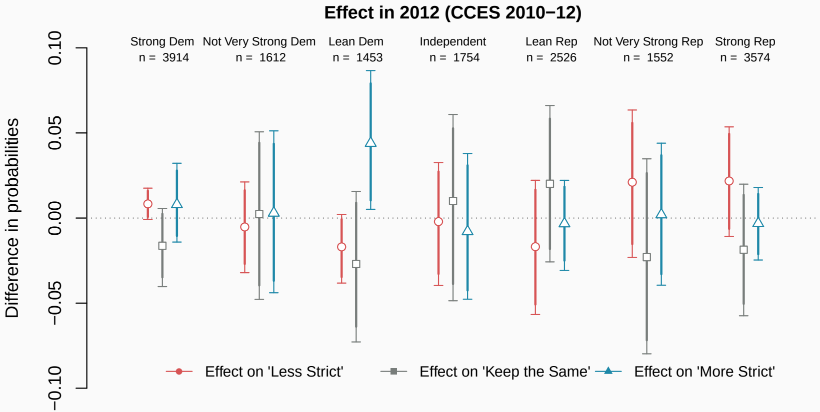
Outcome Distributions by Sub-Group

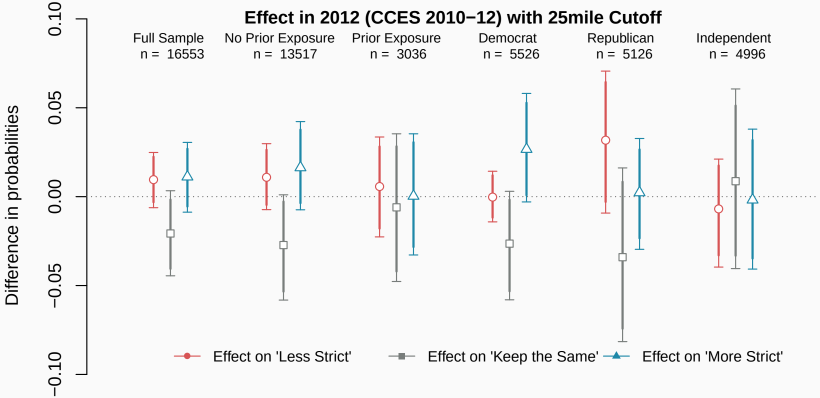




Party ID based on 7-point Scale Measure



Different Distance Threshold: 25 Miles



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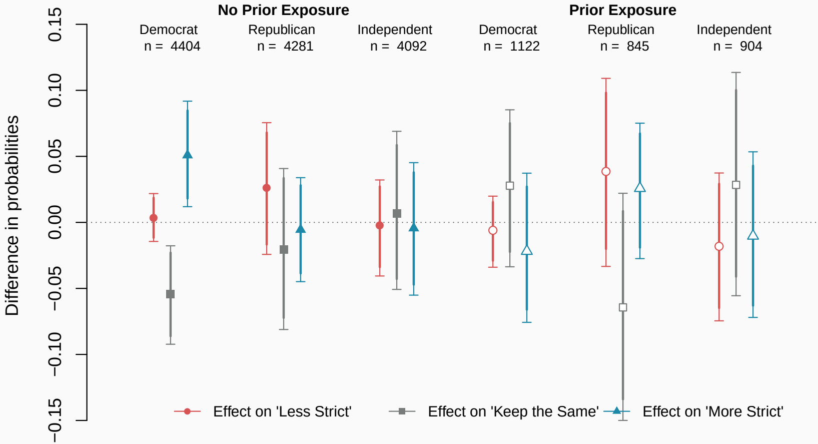
Different Distance Threshold: 25 Miles



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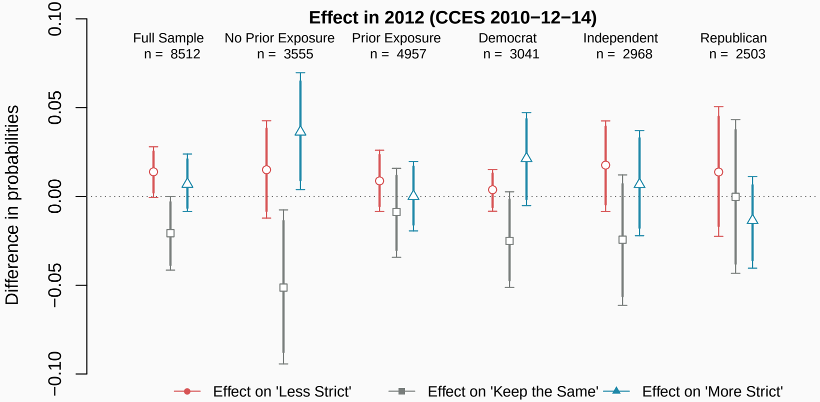
Three-wave Sub-sample: Effect in 2012



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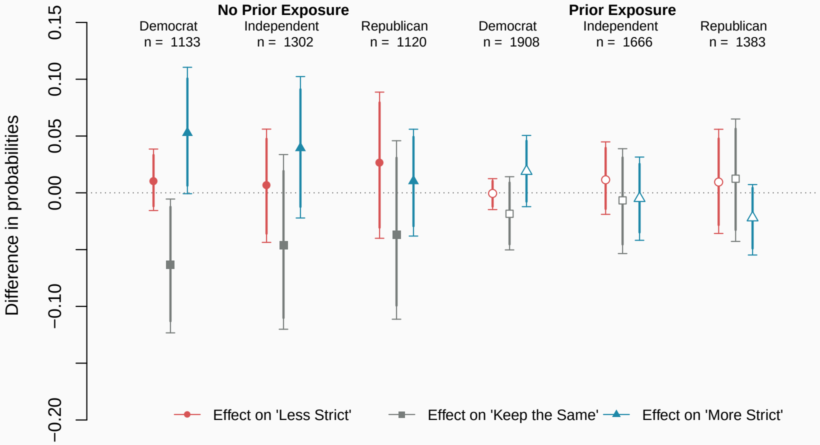
Three-wave Sub-sample: Effect in 2012



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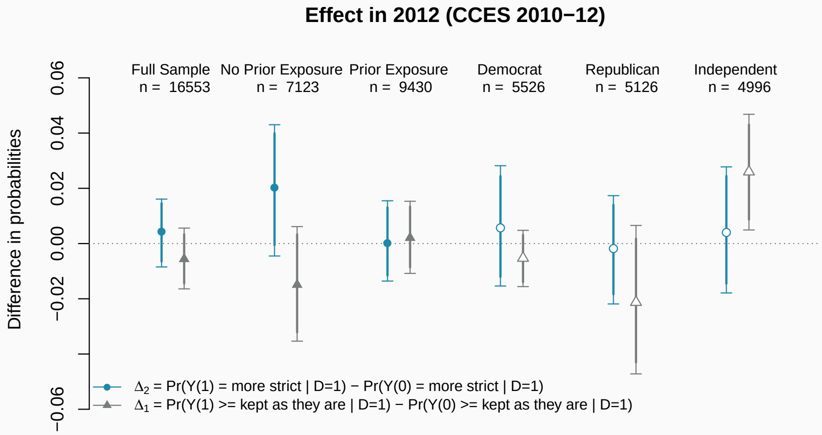
Cumulative Effect: Two-wave Panel

$$\Delta_j = \Pr(Y_{i1}(1) \geq j \mid D_i = 1) - \Pr(Y_{i1}(0) \geq j \mid D_i = 1)$$



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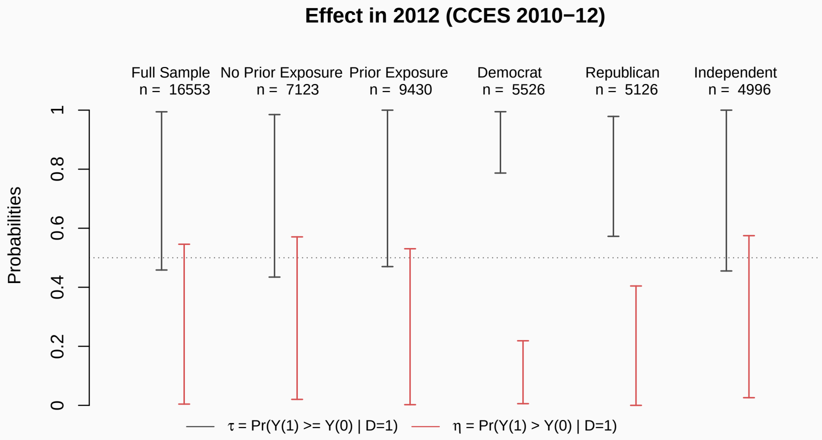
Bound Results: Two-wave Panel

$$\tau = \Pr(Y_{i1}(1) \geq Y_{i1}(0) \mid D_i = 1), \quad \text{and} \quad \eta = \Pr(Y_{i1}(1) > Y_{i1}(0) \mid D_i = 1)$$



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Definition of Mass Shootings

Cases involving the following:

1. Firearms as the primary weapon used,
2. Attacks on non-family members of the general public
3. Attacks in which at least three or more individuals were injured or killed

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